MANDATORY COURSES

First Year, Fall term

M1. Basic Algebra 1

M2. Real Analysis

M3. Probability

Skills: Latex for Mathematicians

First Year, Winter term

M4. Basic Algebra 2

M5. Complex Function Theory

M6. Functional Analysis and Differential Equations

Forms of assessment for mandatory courses: weekly homework, midterm, final

Mandatory Courses Syllabi

M1. BASIC ALGEBRA 1

Course coordinator: Pal Hegedus

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: linear algebra, introductory abstract algebra

Course Level: introductory MS

Brief introduction to the course:

Basic concepts and theorems are presented. Emphasis is put on familiarizing with the aims and methods of abstract algebra. Interconnectedness is underlined throughout. Applications are presented.

The goals of the course:

One of the main goals of the course is to introduce students to the most important concepts and fundamental results in abstract algebra. A second goal is to let them move confidently between abstract and concrete phenomena.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Groups: permutations groups, orbit-stabilizer theorem, cycle notation, conjugation, conjugacy classes of S_n, odd/even permutations,
2. commutator subgroup, free groups, geerators and relations, Dyck's theorem,
3. solvable and simple groups, simplicity of A_n, classical linear groups,
4. Polynomials: Euclidean Algorithm, uniqueness of factorisation, Gauss Lemma, cyclotomic polynomials,
5. polynomials in several variables, homogeneous polynomials, symmetric polynomials, formal power series, Newton's Formulas,
6. Sturm's Theorem on the number of real roots of a polynomial with real coefficients.
7. Rings and modules: simplicity of matrix rings, quaternions, Frobenius Theorem, Wedderburn's Theorem,
8. submodules, homomorphisms, direct sums of modules, free modules,
9. chain conditions, composition series.
10. Partially ordered sets and lattices: Hasse-diagram, chain conditions, Zorn Lemma, lattices as posets and as algebraic structures,
11. modular and distributive lattices, modularity of the lattice of normal subgroups, Boolean algebras, Stone Representation Theorem.
12. Universal algebra: subalgebras, homomorphisms, direct products, varieties, Birkhoff Theorem.

Optional topics:

Resultants, polynomials in non-commuting variables, twisted polynomials, subdirect products, subdirectly irreducible algebras, subdirect representation. Categorical approach: products, coproducts, pullback, pushout, functor categories, natural transformations, Yoneda lemma, adjointfunctors.

References:

1. P J Cameron, Introduction to Algebra, Oxford University Press, Oxford, 2008.
2. N Jacobson, Basic Algebra I-II, WH Freeman and Co., San Francisco, 1974/1980.
3. I M Isaacs, Algebra, a graduate course, Brooks/Cole Publishing Company, Pacific Grove, 1994

M2. REAL ANALYSIS

Course coordinator: Laszlo Csirmaz

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: Undergraduate calculus, Elementary Linear Algebra

Course Level: introductory MS

Brief introduction to the course:

Introduction to Lebesgue integration theory; measure, σ-algebra, σ-finite measures. Different notion of convergences; product spaces, signed measure, Radon-Nikodym derivative, Fubini and Riesz theorems; Weierstrass approximation theorem. Solid foundation in the Lebesgue integration theory, basic techniques in analysis. It also enhances student's ability to make their own notes.At the end of the course students are expected to understand the difference between ”naive” and rigorous modern analysis. Should have a glimpse into the topics of functional analysis as well. They must know and recall the main results, proofs, definition.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Real Analysis.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general. At the end of the course students are expected to understand the difference between ”naive” and rigorous modern analysis. They should have a glimpse into the topics of functional analysis as well. They must know and recall the main results, proofs, definition.

More detailed display of contents (week-by-week):

1. Outer measure, measure, σ-algebra, σ-finite measure. liminf and limsup of sets; their measure. The Borel-Cantelli lemma. Complete measure
2. Caratheodory outer measure on a metric space. Borel sets. Lebesgue measure. Connection between Lebesgue measurable sets and Borel sets

3. Measurable functions. Measurable functions are closed under addition and multiplication. Continuous functions are measurable. Example where the composition of measurable functions is not measurable
4. Limits of measurable functions, sup, inf, lim sup, lim inf. Egoroff's theorem: if f_i converges pointwise a.e to f then it converges uniformly with an exceptional set of measure $<\varepsilon$. Convergence in measure; pointwise convergence for a subsequence.
5. Lusin's theorem: a Lebesgue measurable function is continuous with an exceptional set of measure $<\varepsilon$. Converging to a measurable function by simple functions.
6. Definition of the integral; conditions on a measurable function to be integrable. Fatou's lemma, Monotone Convergence Theorem; Lebesgue's Dominated Convergence Theorem. Counterexample: a sequence of functions tends to f, but the integrals do not converge to the integral of f.
7. Hölder and Minkowsi inequalities; L^p is a normed space. Riesz-Fischer theorem: L^p is complete, conjugate spaces, basic properties
8. Signed measure, absolute continuity, Jordan and Hahn decomposition. Radon-Nikodym derivative. Product measure, Fubini's theorem. Counterexample where the order of integration cannot be exchanged
9. Example for a continuous, nowhere differentiable function. Example for a strictly increasing function which has zero derivative a.e.
10. An increasing function has derivative a.e.
11. Weierstrass' approximation theorem
12. Basic properties of convolution

References:

Online material is available at the following sites:

1. http://www.indiana.edu/~mathwz/PRbook.pdf,
2. http://compwiki.ceu.hu/mediawiki/index.php/Real_analysis

M3. PROBABILITY

Course Coordinator: Gabor Pete

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: basic probability

Course Level: Introductory MS

Brief introduction to the course:

The course introduces the fundamental tools in probability theory.

The goals of the course:

The main goal of the course is to learn fundamental notions like Laws of Large Numbers, martingales, and Large Deviation Theorems.

The learning outcomes of the course:

By the end of the course, students are enabled expertson the topic of the course. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

Week 1 Review of basic notions of probability theory. Famous problems and paradoxes.

Week 2-3 Probabilistic methods in combinatorics. Second moment method, Lovasz Local Lemma.

Week 4 Different types of convergence for random variables.Borel-Cantelli lemmas.

Week 5-6 Laws of Large Numbers. The method of characteristic functions in proving weak convergence: the Central Limit Theorem.

Week 7 Basics of measure-theoretic probability, including conditional expectation with respect to a sub-sigma-algebra.

Week 8 Martingales. Some martingale convergence and optional stopping theorems.

Week 9 Galton-Watson branching processes. Asymptotic results. Birth and death process.

Week 10 Some large deviation theorems, Azuma's inequality.

Week 11-12 Random walks on the integers.Construction and basic properties ofBrownian motion.

References:

1. R. Durrett: Probability. Theory and Examples. 4th edition, Cambridge University Press, 2010.
2. D. Williams: Probability with Martingales. Cambridge University Press, 1991.

M4. BASIC ALGEBRA 2

Course coordinator: Pal Hegedus

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Winter Semester

Prerequisites: Basic Algebra 1

Course Level: intermediate MS

Brief introduction to the course:

Further concepts and theorems are presented, like Galois theory, Noetherian rings, Fundamental Theorem of Algebra, Jordan normal form, Hilbert's Theorems. Emphasis is put on difference of questions at different areas of abstract algebra and interconnectedness is underlined throughout. Applications are presented.

The goals of the course:

One of the main goals of the course is to introduce the main distinct areas of abstract algebra and the fundamental results therein. A second goal is to let them move confidently between abstract and concrete phenomena.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Lattices, Posets: Hasse-digram, Zorn Lemma, modular and distributive lattices,
2. Jordan-Dedekind Theorem, Boolean Algebras.
3. Groups: centralizer,normalizer, class equation, p-groups,
4. nilpotent groups, Frattini subgroup, Frattiniargument,
5. direct product, semidirect product, groups of small order.
6. Commutative rings: unique factorization, principal ideal domains, Euclidean domains,
7. finitely generated modules over principal ideal domains, FundamentalTheorem of finite abelian groups, Jordan normal form of matrices,
8. Noetherianrings, Hilbert Basis Theorem, operations with ideals.
9. Fields: algebraic and transcendental extensions, transcendence degree,

10. Splittingfield, algebraic closure, the Fundamental Theorem of Algebra, normal extensions,finite fields, separable extensions,
11. Galois group, Fundamental Theorem of Galois Theory, cyclotomic fields,
12. radical expressions, insolvability of the quinticequation, traces and norms: Hilbert's Theorem,

Optional topics:

Stone Representation Theorem
Krull-Schmidt Theorem
Artin-Schreier theorems, ordered and formally real fields.
Formal power series
Universal algebra: subalgebras, homomorphisms, direct products, varieties, BirkhoffTheorem,subdirect products, subdirectly irreducible algebras, subdirect representation.
Categorical approach: products, coproducts, pullback, pushout, functor categories, natural transformations, Yoneda lemma, adjointfunctors.

References:
1. N Jacobson, Basic Algebra I-II, WH Freeman and Co., San Francisco, 1974/1980.
2. I M Isaacs, Algebra, a graduate course, Brooks/Cole Publishing Company, Pacific Grove, 1994

M5. COMPLEX FUNCTION THEORY

Course coordinator: Karoly Boroczky

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Winter Semester

Prerequisites: Real analysis

Course Level: intermediate MS

Brief introduction to the course:

Fundamental concepts and themes of classic function theory in one complex variable are presented: complex derivative of complex valued functions, contour integration, Cauchy's integral theorem, Taylor and Laurent series, residues, applications, conformal maps, Riemann mapping theorem.

The goals of the course:

The goal of the course is to acquaint the students with the fundamental concepts and results of classic complex function theory.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Complex numbers, complex differentiable functions
Week 2: Power series
Week 3: The exponential and logarithm function
Week 4: Complex line integrals
Week 5: Complex line integrals, primitives
Week 6: Goursat's theorem, Cauchy integral theorem
Week 7: Evaluation of some integrals, Cauchy's integral formulas
Week 8: Theorem on power series development of holomorphic functions, Liouville's theorem, fundamental theorem of algebra, identity theorem
Week 9: Simply connected domains, complex logarithm. Laurent series, isolated singularities, residues, residue theorem
Week 10: Applications of the residue theorem, argument principle, Rouche's theorem, open mapping theorem
Week 11: Fractional linear transformations, conformal maps, Schwarz lemma, automorphisms of the disc and the upper half plane, Vitali-Montel theorem
Week 12: Riemann mapping theorem

References:

1.E. M. Stein-R. Shakarchi: Complex analysis, Princeton Lectures in analysis II, Princeton University Press 2003

2.R.E. Greene-S.G. Krantz: Function theory of one complex variable, Graduate Studies in Mathematics Vol 40, American Mathematical Society, 2002

3. J. B. Conway: Functions of one complex variable I, Springer-Verlag, 1978.

M6. FUNCTIONAL ANALYSIS AND DIFFERENTIAL EQUATIONS

Course coordinator: Gheorghe Morosanu

No. of Credits: 3 and no. of ECTS credits: 6

Time Period of the course: Winter Semester

Prerequisites: Real analysis, Basic algebra 1

Course Level: intermediate MS

Brief introduction to the course:
The basic definitions and results of functional analysis will be presented about Hilbert spaces and Banach spaces including Lp spaces, and applications to problems involving differential equations will be discussed.

The goals of the course:
The main goal of the course is to provide important tools of functional analysis and to illustrate their applicability to the theory of differential equations.

The learning outcomes of the course:
By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: The Hahn-Banach theorems
Week 2: The uniform boundedness principle, the open mapping theorem, and the closed graph theorem
Week 3: Weak topologies. Reflexive and separable spaces
Week 4: L^p spaces, reflexivity, separability
Weeks 5-6: Hilbert space theory
Week 7: Test functions on (a,b), $W^{1,p}$(a,b)
Week 8: Linear differential equations in distributions
Week 9: Variational approach to boundary value problems for second order differential equations
Week 10: Bounded and unbounded linear operators
Weeks 11-12: Uniformly continuous and strongly continuous linear semigroups and applications to boundary value problems associated with the heat and wave equations

Reference:
H. Brezis, *Functional Analysis, SobolevSpaces and Partial Differential Equations,* Springer, 2011.

LATEX for MATHEMATICIANS

Course coordinator: Laszlo Csirmaz

No. of Credits: 1 and no. of ECTS credits: 2

Time Period of the course: Fall Semester

Prerequisites: none

Course Level: introductory MS

Brief introduction to the course:
The short intensive course introduces the LaTeX typesetting language to all prospective interested students. It is used for writing all scientific papers and presentations in Mathematics, this the knowledge how to use it is essential for all to be professionals.

The goals of the course:
The course aims to teach the basic features of LaTeX along its usage inside CEU. By attending the course students should acquire all necessary skills to be able to prepare a moderate scientific paper and a short mathematical presentation using LaTeX.

The learning outcomes of the course:
By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

- Basic document layout
- Sectioning: section, subsection, unnumbered sections
- Fonts, typefaces, sizes
- Environments: numbered and bulleted list, quotation, centered and justified display, verse, verbatim
- Equation, pictures, tables, tabular material
- Floating material
- Macros, conditional macros, tricks with macros. Typesetting mathematics: accents, mathematical fonts, alignment in mathematical expressions and formulas, nested arrays
- The overall structure of a document: document classes
- Bibliography, using Bibtex
- Making a presentation: the *beamer* class
- TeX usage in CEU, distribution, packages
- Thesis writing in LaTeX, the ceuthesis package

Reference:
Learning Latex, D.F.Griffits, D.J.Higham, SIAM, Philadelpha, 1997

MS ELECTIVE COURSES
Suggested form of assessment for

- *elective live courses*: regular homework, and presentation or final
- *elective reading courses*: regular homework

THEORY of ALGORITHMS
APPLIED PARTIAL DIFFERENTIAL EQUATIONS
EVOLUTION EQUATIONS AND APPLICATIONS
CONTROL OF DYNAMIC SYSTEMS
NON-STANDARD ANALYSIS
SPECIAL FUNCTIONS AND RIEMANN SURFACES
DIFFERENTIAL GEOMETRY
SMOOTH MANIFOLDS AND DIFFERENTIAL TOPOLOGY
STOCHASTICS PROCESSES AND APPLICATIONS
PROBABILITY 2
MATHEMATICAL STATISTICS
MULTIVARIATE STATISTICS
INFORMATION THEORY
INFORMATION DIVERGENCES IN STATISTICS
NONPARAMETRIC STATISTICS
TOPICS IN FINANCIAL MATHEMATICs
QUANTITATIVE FINANCIAL RISK ANALYSIS
RISK MEASURES
BIOINFORMATICS
MATHEMATICAL MODELS IN BIOLOGY AND ECOLOGY
EVOLUTIONARY GAME THEORY AND POPULATION DYNAMICS
PROBABILISTIC MODELS OF THE BRAIN AND THE MIND
ERGODIC THEORY
MATHEMATICAL METHODS IN STATISTICAL PHYSICS
FRACTALS AND DYNAMICAL SYSTEMS
COMPUTATIONAL NUMBER THEORY
COMPUTATIONS IN ALGEBRA
MATRIX COMPUTATIONS WITH APPLICATIONS
CRYPTOGRAPHIC PROTOCOLS
CRYPTOLOGY
COMBINATORIAL OPTIMIZATION
NONLINEAR OPTIMIZATION
TOPICS IN NONLINEAR OPTIMIZATION
OPTIMIZATION IN ECONOMICS
INTRODUCTION TO DISCRETE MATHEMATICS
GRAPH THEORY AND APPLICATIONS
PACKING AND COVERING
CONVEX POLYTOPES
COMBINATORIAL GEOMETRY
GEOMETRY OF NUMBERS

EXACT AND APPROXIMATE ALGORITHMS FOR VEHICLE ROUTING PROBLEMS
MATHEMATICAL FINANCE: A PRIMER FOR PRACTITIONERS
SCIENTIFIC PYTHON
HOW TO GAMBLE IF YOU MUST

SYLLABI of ELECTIVE COURSES

THEORY OF ALGORITHMS

Course coordinator: Istvan Miklos

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: -

Course Level: introductory MS

Brief introduction to the course:

Greedy and dynamic programming algorithms. Famous tricks in computer science. The most important data structures in computer science. The Chomsky hierarchy of grammars, parsing of grammars, relationship to automaton theory.Computers, Turing machines, complexity classes P and NP, NP-complete. Stochastic Turing machines, important stochastic complexity classes. Counting classes, stochastic approximation with Markov chains.

The goals of the course:

To learn dynamic programming algorithms, the most important data structures like chained lists, hashing, etc., and the theoretical background of computer science (Turing machines, complexity classes). To get an overview of standard tricks in algorithm design, and an introduction in stochastic computing.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1.
Theory: The O, Ω and Θ notations. Greedy and dynamic programming algorithms.Kruskal's algorithm for minimum spanning trees, the folklore algorithm for the longest common subsequence of two strings.
Practice: The money change problem and other famous dynamic programming algorithms

Week 2.

Theory: Dijstra’s algorithm and other algorithms for the shortest path problem.
Practice: Further dynamic programming algorithms.

Week 3.
Theory: Divide-and-conqueror and checkpoint algorithms. The Hirshberg’s algorithm for aligning sequences in linear space
Practice: Checkpoint algorithms. Reduced memory algorithms.

Week 4.
Theory: Quick sorting. Sorting algorithms.
Practice: Recursive functions. Counting with inclusion-exclusion.

Week 5.
Theory: The Knuth-Morrison-Pratt algorithm. Suffix trees.
Practice: String processing algorithms. Exact matching and matching with errors.

Week 6.
Theory: Famous data structures. Chained lists, reference lists, hashing.
Practice: Searching in data structures.

Week 7.
Theory: The Chomsky-hierarchy of grammars. Parsing algorithms.Connections to the automaton theory.
Practice: Regular expressions, regular grammars. Parsing of some special grammars between regular and context-free and between context-free and context-dependent classes.

Week 8.
Theory: Introduction to algebraic dynamic programming and the object-oriented programming.
Practice: Algebraic dynamic programming algorithms.

Week 9.
Theory: Computers, Turing-machines, complexity and intractability, complexity of algorithms, the complexity classes P and NP. 3-satisfiability, and NP-complete problems.
Practice: Algorithm complexities. Famous NP-complete problems.

Week 10.
Theory: Stochastic Turing machines. The complexity class BPP. Counting problems, #P, #P-complete, FPRAS.
Practice: Stochastic algorithms.

Week 11.
Theory: Discrete time Markov chains. Reversible Markov chains, Frobenius theorem. Relationship between the second largest eigenvalue modulus and convergence of Markov chains.Upper and lower bounds on the second largest eigenvalue.
Practice: Upper and lower bounds on the second largest eigenvalue.

Week 12.
Theory: The Sinclair-Jerrum theorem: relationship between approximate counting and sampling.
Practice: Some classical almost uniform sampling (unrooted binary trees, spanning trees).

Reference:
Dasgupta-Papadimitriuo-Vazirani: Algorithms, http://www.cs.berkeley.edu/~vazirani/algorithms/all.pdf

APPLIED PARTIAL DIFFERENTIAL EQUATIONS

Lecturer: Gheorghe Morosanu
No. of Credits : 3and no. of ECTS credits: 6
Prerequisites:linear algebra, real and complex analysis
Course Level: introductory MS

Brief introduction to the course:
The main classes of partial differential equations will be discussed and some applications to specific problems will be investigated.

The goals of the course:
The main goals of the course is to provide the most important methods of the theory of partial differential equations and to solve specific examples.

The learning outcomes of the course:
By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Various models involving linear and nonlinear partial differential equations

2. Elliptic equations. Maximum principles

3-4. Variational solutions for elliptic boundary value problems. Examples

5-6. Parabolic equations. Applications

6-7. Hyperbolic equations and systems. Vibrating strings and membranes

8-10. Theory for nonlinear partial differential equations. Variational and nonvariational techniques. Applications

11. Conservation laws

12. Laplace transform solution of partial differential equations

References:

1. L.C. Evans, Partial Differential Equations, Graduate Studies in Math. 19, AMS, Providence, Rhode Island, 1998.
2. R. Haberman, Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, Fourth Edition, Pearson Education, Inc. Pearson Prentice Hall, 2004.
3. R.M.M. Mattheij, S.W. Rienstra and J.H.M. ten ThijeBoonkkamp, Partial Differential Equations. Modeling, Analysis, Computation, SIAM, Philadelphia, 2005.

EVOLUTION EQUATIONS AND APPLICATIONS

Lecturer: Gheorghe Morosanu
No. of Credits: 3and no. of ECTS credits: 6
Prerequisites:linear algebra, real and complex analysis, functional analysis
Course Level: introductory MS

Brief introduction to the course:
Some basic existence results on evolution equations will be presented and applications to problems involving differential equations will be discussed.

The goals of the course:
The main goal of the course is to provide important results on abstract evolution equations and to illustrate their applicability to specific examples.

The learning outcomes of the course:
By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):
1-2. Preliminaries of linear and nonlinear functional analysis
3-4. Existence, uniqueness and regularity of solutions to evolution equations in Hilbert spaces
5. Boundedness of solutions on the positive half axis, weak convergence of averages as t goes to infinity
6-7. Stability of solutions. Strong and weak convergence results
8. Periodic forcing. The asymptotic dosing problem
9-12. Applications to delay equations, parabolic and hyperbolic boundary value problems. Specific examples.

References:

1. H. Brezis, Operateursmaximaux monotones et semigroupes de contractions dans les espaces de Hilbert, North Holland, Amsterdam, 1973.

2. V.-M. Hokkanen and G. Morosanu, Functional Methods in Differential Equations, Chapman & Hall/CRC, 2002.
3. G. Morosanu, Nonlinear Evolution Equations and Applications, Reidel, 1988.

CONTROL OF DYNAMIC SYSTEMS

Course coordinator: Gheorghe Morosanu

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Real Analysis, Ordinary Differential Equations

Course Level: advanced MS

Brief introduction to the course:

Basic principles and methods of control theory are discussed. The main concepts are observability, controllability, stabilizability, optimality conditions, etc.) are addressed, with special emphasis on linear differential systems and quadratic functionals. Many applications are discussed in detail. The course is designed for students oriented to Applied Mathematics.

The goals of the course:

The main goal of the course is to introduce students to the theory of optimal control for differential systems. We also intend to discuss specific problems which arise from down-to-earth applications in order to illustrate this remarkable theory.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Linear Differential Systems (existence of solutions, variation of constants formula, continuous dependence of solutions on data, exercises)

Week 2: Nonlinear Differential Systems (local and global existence of solutions for the Cauchy problem, continuous dependence on data, differential inclusions, exercises)

Week 3: Basic Stability Theory (concepts of stability, stability of the equilibrium, stability by linearization, Lyapunov functions, applications)

Week 4: Observability of linear autonomous systems (definition, observability matrix, necessary an sufficient conditions for observability, examples)

Week 5: Observability of linear time varying systems (definition, observability matrix, numerical algorithms for observability, examples)

Week 6: Input identification for linear systems (definition, the rank condition in the case of autonomous systems, examples)

Week 7: Controllability of linear systems (definition, controllability of autonomous systems, controllability matrix, Kalman's rank condition, the case of time varying systems, applications)

Week 8: Controllability of perturbed systems (perturbations of the control matrix, nonlinear autonomous systems, time varying systems, examples)

Week 9: Stabilizability (definition, state feedback, output feedback, applications)

Week 10: Introduction to optimal control theory (Meyer's problem, Pontryagin's Minimum Principle, examples)

Week 11: Linear quadratic regulator theory (introduction, the Riccati equation, perturbed regulators, applications)

Week 12: Time optimal control (general problem, linear systems, bang-bang control, applications)

References:

1. N.U. Ahmed, Dynamic Systems and Control with Applications, World Scientific, 2006.
2. E.B. Lee and L. Markus, Foundations of Optimal Control Theory, John Wiley, 1967.

NON-STANDARD ANALYSIS

Course Coordinator: Laszlo Csirmaz

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Complex Functions, Real Analysis, Functional Analysis

Course Level: advanced MS

Brief introduction to the course:

Non-standard analysis is an alternate way to the basic notions of analysis, where the "infinitely small" gets an exact meaning. At the end of the course students will know the notion of enlargement, the distinction between internal and external sets; have exact explanation to the intuitive feeling for uniform convergence, the notion of monad, and characterization of compact topological spaces in terms of nearly standard points. The course culminates in proving the famous Picard's theorem on essential singularities.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Non-standard analysis

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Tools from mathematical logic: first order and higher order theories
2. The compactness theorem; compactness for higher order logic
3. Enlargements, internal and external sets, existence of enlargements
4. Elementary analysis: convergence, uniform convergence, continuous functions, uniformly continuous functions, differentiation
5. Integration, existence of Riemann integrals, main theorem of analysis
6. Dini's theorem, equicontinuous sequence of functions.
7. Topological spaces: compactness, Thichonov's theorem, metrizability.
8. Uhrysson's theorem on metrizable spaces
9. Lacunarypolymoials: theorems of Montel and Kakeya.
10. Complex functions, analytic functions, different topologies on the extension of the complex numbers, their connection to analytic functions
11. Proof of Picard's theorem on the essential singularities of analytic functions.
12. Julia's directions and generalizations.

Reference:

Abraham Robinson, Non-standard Analysis, Princeton Univ. Press, 1995.

SPECIAL FUNCTIONS AND RIEMANN SURFACES

Course coordinator: KarolyBoroczky

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Complex Function Theory

Course Level: intermediate MS

Brief introduction to the course:

Some interesting topics in one complex variable are presented like gamma function, Riemann's zeta function, analytic continuation, monodromy theorem, Riemann surfaces, universal cover, uniformization theorem

The goals of the course:

The goal of the course is to acquaint the students with the basic understanding of special functions and Riemann surfaces

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Analytic continuation, Monodromy Theorem
Week 2: Normal families
Week 3: Blaschke products, The Mittag-Leffler theorem
Week 4: The Weierstrass theorem
Week 5: Euler' Gamma Function
Week 6: Riemann's zeta function
Week 7: Riemann surfaces
Week 8: Simply connected Riemann surfaces, hyperbolic structure on the disc
Week 9: Covering spaces, Universal cover
Week 10: Covering the twice punctured plane, Great Picard theorem
Week 11: Differential forms on Riemann surfaces
Week 12: Overview of uniformization theorem and Riemann-Roch theorem

Reference:

2 J. B. Conway: Functions of one complex variable I and II, Springer-Verlag, 1978.

DIFFERENTIAL GEOMETRY

Course coordinator: BalazsCsikos

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Real Analysis, Basic Algebra 1

Course Level: intermediate MS

Brief introduction to the course:

This course is split into three parts. In the first two parts we give an introduction to the classical roots of modern differential geometry, the theory of curves and hypersurfaces in n-dimensional Euclidean spaces. In the third part foundations of manifold theory are laid.

The goals of the course:

Differential geometry is a powerful combination of geometry and analysis. It has various applications within many branches of mathematics (theory of ordinary and partial differential equations, calculus of variations, algebraic geometry, ...), as well as in mathematical physics, optics, mechanics, engineering, etc. This course gives an introduction to differential geometry, following the historical development of the subject.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Parameterized curves. (Length,reparameterizations, natural reparameterization. Tangent line and osculating affine subspaces.)

Week 2: Frenet theory of curves. (Frenet frame, curvatures, Frenet equations. Fundamental Theorem of curve theory.)

Week 3: Some applications. (Osculating circle, evolute, involute. Envelope of a family of planar curves, and other optional applications.)

Week 4: Hypersurfaces. (Tangent hyperplane, Gauss map. Normal curvature, Meusnier's theorem. Fundamental forms, principal curvatures and principal directions, Euler's formula,Weingarten map, Gaussian and Minkowski curvature.)

Week 5: Applications. (Surfaces of revolution, ruled and developable surfaces, and other optional applications.)

Week 6: Fundamental equations of hypersurface theory. (Gauss and Codazzi-Mainardi equations. Intrinsic geometry of a hypersurface, TheoremaEgregium.)

Week 7: The Gauss-Bonnet formula. (Integration on hypersurfaces, geodesic curvature of curves on a hypersurface, local and global versions of the Gauss-Bonnet formula.)

Week 8:Differentiable manifolds. (Definitions. Examples, submanifolds of a manifold. Smooth maps. Tangent vectors of a manifold. The derivative of a smooth map.)

Week 9: Lie algebra of vector fields. (Definition and properties of the Lie bracket, the flow generated by a vector field. Geometrical meaning of the Lie bracket.)

Week 10: Connections. (Definition. Christoffel symbols with respect to a chart. Torsion. Parallel transport. Compatibility with a Riemannian metric. Levi-Civita connection.)

Week 11: Curvature tensor. (Definiton. Linearity over smooth functions. Symmetry properties. Derived curvature quantities: sectional curvature, Ricci curvature, scalar curvature.

Week 12: Geodesics. (Definition. Exponential map. Normal coordinates. Gauss lemma. Formula for the first variation of the length. Short geodesic segments minimize the length.)

Reference:

B. Csikos: Differential Geometry (http://www.cs.elte.hu/geometry/csikos/dif/dif.html)

SMOOTH MANIFOLDS AND DIFFERENTIAL TOPOLOGY

Lecturer: Andras Nemethi

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: -

Course Level: intermediate MS

Brief introduction to the course:

Basic principles and methods concerning differentiable manifolds and differentaible maps are discussed. The main concepts (submersions, transversality, smooth manifolds and manifolds with boundary, orientation, degree and intersection theory, etc.) are addressed, with special emphasis on different connections with algebraic topology (coverings, homological invariants). Many applications are discussed in detail (winding number, Borsuk-Ulam theorem, Lefschetz fixed point theory, and different connections with algebraic geometry).

The course is designed for students oriented to (algebraic) topology or algebraic geometry.

The goals of the course:

The main goal of the course is to introduce students to the theory of smooth manifolds and their invariants. We also intend to discuss different connections with algebraic topology, (co)homology theory and complex/real algebraic geometry.

The learning outcomes of the course:

The students will learn important notions and results in theory of smooth manifolds and smooth maps. They will meet the first non-trivial invariants in the classification of maps and manifolds. They will gain crucial skills and knowledge in several parts of modern mathematics. Via the exercises, they will learn how to use these tools in solving specific topological problems.

More detailed display of contents:

Week 1: Derivatives and tangents (definitions, inverse function theorem, immersions).

Week 2: Submersions (definitions, examples, fibrations, Sard's theorem, Morse functions).

Week 3: Transversality (definitions, examples, homotopy and stability).

Week 4: Manifolds and manifolds with boundary (definition, examples, one-manifolds and consequences).

Week 5: Vector bundles (definition, examples, tangent bundles, normal bundles, compex line bundles).

Week 6: Intersection theory mod 2 (definition, examples, winding number, Borsuk-Ulam theorem).

Week 7: Orientation of manifolds (definition, relation with coverings, orientation of vector bundles, applications).

Week 8: The degree (definition, examples, applications, the fundamental theorem of algebra, Hopf degree theorem).

Week 9: Oriented intersection theory (definitions, examples, applications, connection with homology theory).

Week 10: Lefschetz fixed-point theorem (the statement, examples).

Week 11: Vector fields (definition, examples, the index of singular points).

Week 12: Poincare-Hopf theorem (the Euler characteristic, discussion, examples).

References:

1.John W. Milnor, Topology from the Differentiable Viewpoint, Princeton Landmarks in Mathematics, Princeton University Press.

2. Victor Guillemin and Alan Pollack, Differential Topology.

STOCHASTICS PROCESSES AND APPLICATIONS

Course Coordinator: Gabor Pete

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites:basic probability

Course Level:introductory MS

Brief introduction to the course:

The most common classes of stochastic processes are presented that are important in applications an stochastic modeling. Several real word applications are shown. Emphasis is put on learning the methods and the tricks of stochastic modeling.

The goals of the course:

The main goal of the course is to learn the basic tricks of stochastic modeling via studying many applications. It is also important to understand the theoretical background of the methods.

The learning outcomes of the course:

The students are experts on the topic of the course. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

1. Stochastic processes: Kolmogorov theorem, classes of stochastic processes, branching processes
2. Poisson processes: properties, arrival times; compound, non-homogeneous and rarefied Poisson process; application to queuing
3. Martingales: conditional expectation, martingales, stopping times, Wald's equation, convergence of martingales
4. Applications of martingales: applications to risk processes, log-optimal portfolio

5. Martingales and Barabási-Albert graph model: preferential attachment (BA model), degree distribution
6. Renewal processes: renewal function, renewal equation, limit theorems, Elementary Renewal Theorem,
7. Renewal processes: Blackwell's theorem, key renewal theorem, excess life and age distribution, delayed renewal processes
8. Renewal processes: applications to queuing, renewal reward processes, age dependent branching process
9. Markov chains: classification of states, limit theorems, stationary distribution
10. Markov chains: transition among classes, absorption, applications
11. Coupling: geometrically ergodic Markov chains, proof of renewal theorem
12. Regenerative processes: limit theorems, application to queuing, Little's law

References:

1. S. M. Ross, Applied Probability Models with Optimization Applications, Holden-Day, San Francisco, 1970.
2. S. Asmussen, Applied Probability and Queues, Wiley, 1987.

PROBABILITY 2

Course Coordinator: Gabor Pete

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Probability 1

Course Level:advanced MS

Brief introduction to the course:

The course introduces advanced tools about martingales, random walks and ergodicity.

The goals of the course:

The main goal of the course is to learn fundamental notions like Laws of Large Numbers, martingales, and Large Deviation Theorems.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

Week 1-2 Martingales. Optional stopping theorems. Maximal inequalities.Martingale convergence theorems.

Week 3-4 Processes with independent increments. Brownian motion. Lévyprocesses. Stable processes. Bochner-Khintchine theorem.

Week 5 Markovprocesses. Infinitesimal generator. Chapman-Kolmogorov equations.

Week 6-7 Random walks on graphs, Markov chains, electric networks.

Week 8-9 Recurrence,ergodicity, existence of stationary distribution, mixing times.

Week 10 Pólya's theorem on random walks on the integer lattice.

Week 11 Ergodic theory of stationary processes. von Neumann and Birkhoffergodic theorems.

Week 12 Central limit theorem for martingales and for Markov processes.

References:

1. R. Durrett: Probability. Theory and Examples. 4th edition, Cambridge University Press, 2010.
2. D. Williams: Probability with Martingales. Cambridge University Press, 1991.
3. W. Feller: An Introduction to Probability Theory and its Applications, Vol. II., Second edition. Wiley, New York , 1971.

MATHEMATICAL STATISTICS

Course Coordinator: Marianna Bolla

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites:basic probability

Course Level:introductory MS

Brief introduction to the course:

While probability theory describes random phenomena, mathematical statistics teaches us how to behave in the face of uncertainties, according to the famous mathematician Abraham Wald. Roughly speaking, we will learn strategies of treating randomness in everyday life. Taking this course is suggested between the Probability and Multivariate Statistics courses.

The goals of the course:

The course gives an introduction to the theory of estimation and hypothesis testing. The main concept is that our inference is based on a randomly selected sample from a large population, and hence, our observations are treated as random variables. Through the course we intensively use facts and theorems known from probability theory, e.g., the laws of large numbers. On this basis, applications are also discussed, mainly on a theoretical basis, but we make the students capable of solving numerical exercises.

The learning outcomes of the course:

Students will be able to find the best possible estimator for a given parameter by investigating the bias, efficiency, sufficiency, and consistency of an estimator on the basis of theorems and theoretical facts. Students will gain familiarity with basic methods of estimation and will be able to construct statistical tests for simple and composite hypotheses. They will become familiar with applications to real-world data and will be able to choose the most convenient method for given real-life problems.

More detailed display of contents:

1. Statistical space, statistical sample. Basic statistics, empirical distribution function, Glivenko-Cantelli theorem.
2. Descriptive study of data, histograms. Ordered sample, Kolmogorov-Smirnov Theorems.
3. Sufficiency, Neyman-Fisher factorization. Completeness, exponential family.
4. Theory of point estimation: unbiased estimators, efficiency, consistency.
5. Fisher information. Cramer-Rao inequality, Rao-Blackwellization.
6. Methods of point estimation: maximum likelihood estimation (asymptotic normality), method of moments, Bayes estimation. Interval estimation: confidence intervals.
7. Theory of hypothesis testing, Neyman-Pearson lemma for simple alternative and its extension to composite hypotheses.
8. Parametric inference: z, t, F, chi-square, Welch, Bartlett tests.
9. Nonparametric inference: chi-square, Kolmogorov-Smirnov, Wilcoxon tests.
10. Sequential analysis, Wald-test, Wald-Wolfowitz theorem.
11. Two-variate normal distribution and common features of methods based on it. Theory of least squares, regression analysis, correlation, Gauss-Markov Theorem.
12. One-way analysis of variance and analyzing categorized data.

References:

1. C.R. Rao, Linear statistical inference and its applications. Wiley, New York, 1973.

2. G. K. Bhattacharyya, R. A. Johnson, Statistical concepts and methods. Wiley, New York, 1992.

3. C. R. Rao, Statistics and truth. World Scientific, 1997.

Handouts: tables of notable distributions (parameters and quantile values of the distributions).

MULTIVARIATE STATISTICS

Course Coordinator: Marianna Bolla

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Mathematical Statistics

Course Level: intermediate MS

Brief introduction to the course:

The course generalizes the concepts of Mathematical Statistics to multivariate observations and multidimensional parameter spaces. Students will learn basic models and methods of supervised and unsupervised learning together with applications to real-world data.

The goals of the course:

The first part of the course gives an introduction to the multivariate normal distribution and deals with spectral techniques to reveal the covariance structure of the data. In the second part methods for reduction of dimensionality will be introduced (factor analysis and canonical correlation analysis) together with linear models, regression analysis and analysis of variance. In the third part students will learn methods of classification and clustering to reveal connections between the observations, and get insight into some modern algorithmic models. Applications are also discussed, mainly on a theoretical basis, but we make the students capable of interpreting the results of statistical program packages.

The learning outcomes of the course:

Students will be able to find the best possible estimator for a given parameter by investigating the bias, efficiency, sufficiency, and consistency of an estimator on the basis of theorems and theoretical facts. Students will gain familiarity with basic methods of estimation and will be able to construct statistical tests for simple and composite hypotheses. They will become familiar with applications to real-world data and will be able to choose the most convenient method for given real-life problems.

More detailed display of contents:

1. Multivariate normal distribution, conditional distributions, multiple and partial correlations.
2. Multidimensional central limit theorem. Multinomial sampling and deriving the asymptotic distribution of the chi-square statistics.
3. Maximum likelihood estimation of the parameters of a multivariate normal

population. The Wishart distribution.
4. Fisher-information matrix. Cramer-Rao and Rao-Blackwell-Kolmogorov theorems for multivariate data and multidimensional parameters.
5. Likelihood ratio tests and testing hypotheses about the multivariate normal mean.
6. Comparing two treatments. Mahalanobis D-square and the Hotelling's T-square distribution.
7. Multivariate statistical methods for reduction of dimensionality: principal component and factor analysis, canonical correlation analysis.
8. Theory of least squares. Multivariate regression, Gauss-Markov theory.
9. Fisher-Cochran theorem. Two-way analysis of variance, how to use ANOVA tables.
10. Classification and clustering. Discriminant analysis, k-means and hierarchical clustering methods.
11. Factoring and classifying categorized data. Contingency tables, correspondence analysis.
12. Algorithmic models: EM-algorithm for missing data, ACE-algorithm for generalized regression, Kaplan-Meier estimates for censored observations.

References:

1. K.V. Mardia, J.T. Kent, and M. Bibby, Multivariate analysis. Academic Press, New York, 1979.
2. C.R. Rao, Linear statistical inference and its applications. Wiley, New York, 1973.

INFORMATION THEORY

Course Coordinator: Laszlo Gyorfi

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Probability 1

Course Level: intermediate MS

Brief introduction to the course:

The course summarizes the main principles of information theory: data compression (lossless source coding), quantization (lossy source coding), optimal decisions, channel coding.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of the Information Theory.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1-2 Definition and formal properties of Shannon's information measures

Week 3-4 Source and channel models. Source coding, block and variable length codes, entropy rate. Arithmetic codes. The concept of universal coding.

Week 5-6 Channel coding (error correction), operational definition of channel capacity. The coding theorem for discrete memoryless channels. Shannon's source-channel transmission theorem.

Week 7-8 Outlook to multiuser and secrecy problems.

Week 9-10 Exponential error bounds for source and channel coding. Compound and arbitrary varying channels. Channels with continuous alphabets; capacity of the channel with additive Gaussian noise.

Week 11-12 Elements of algebraic coding theory; Hamming and Reed-Solomon codes.

References:

1. T.M. Cover & J.A. Thomas: Elements of Information Theory. Wiley, 1991.

2. I. Csiszar& J. Korner: Information Theory. Academic Press, 1981.

INFORMATION DIVERGENCES IN STATISTICS

Course coordinator: Laszlo Gyorfi

No. of Credits: 3 and no. of ECTS credits: 6

Prerequisities: Probability 1

Course Level:intermediate MS

Brief introduction to the course:

The course summarizes the main principles of decision theory and hypotheses testing: simple and composite hypotheses, L1 distance, I-divergence, large deviation, robust detection, testing homogeneity, testing independence.

The goals of the course:

To become familiar with the notion of Information Divergences in Statistics.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

Week 1. Bayes decision.
Week 2. Testing simple hypotheses.
Week 3. Repeated observations.
Week 4. Total variation and I-divergence.
Week 5. Large deviation of L1 distance.
Week 6. L1-distance-based strong consistent test for simple versus composite hypotheses.
Week 7. I-divergence-based strong consistent test for simple versus composite hypotheses.
Week 8. Robust detection.
Week 9-10. Testing homogeneity.
Week11-12. Testing independence.

Reference:
http://www.cs.bme.hu/~gyorfi/testinghi.pdf

NONPARAMETRIC STATISTICS

Course coordinator: Laszlo Gyorfi

No. of Credits: 3 and no. of ECTS credits: 6

Prerequisities: Probability 1

Course Level:intermediate MS

Brief introduction to the course:

The course summarizes the main principles of nonparametric statistics: nonparametric regression estimation, pattern recognition, prediction of time series, empirical portfolio selection, nonparametric density estimation.

The goals of the course:

To learn the main methods of Nonparametric Statistics.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

Week 1. Regression problem, L_2 error.
Week 2. Partitioning, kernel, nearest neighbor estimate.
Week3. Prediction of stationary processes.
Week4. Machine learning algorithms.
Week5. Bayes decision, error probability.
Week6. Pattern recognition, partitioning, kernel, nearest neighbor rule.
Week7. Portfolio games, log-optimality.
Week8. Empirical portfolio selection.
Week9-10. Density estimation, L_1 error.
Week11-12. Histogram, kernel estimate.

References:

1. http://www.cs.bme.hu/~oti/portfolio/icpproject/ch5.pdf
2. http://www.cs.bme.hu/~oti/portfolio/icpproject/ch2.pdf

TOPICS IN FINANCIAL MATHEMATICS

Course coordinator: Miklos Rasonyi

No. of Credits: 3 and no. of ECTS credits: 6

Prerequisities: Probability 1

Course Level: intermediate MS

Brief introduction to the course:

Basic concepts of stochastic calculus with respect to Brownian motion. Martingales, quadratic variation, stochastic differential equations. Fundamentals of continuous-time mathematical finance; pricing, replication, valuation using PDE methods. Exotic options, jump processes.

The goals of the course:

To obtain a solid base for applying continuous-time stochastic finance techniques; a firm knowledge of basic notions, methods. An introduction to most often used models.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents:

Week 1. From random walk to Brownian motion. Quadratic variation.

Week 2. Ito integral, Ito processes. Ito's formula and its applications.

Week 3. Stochastic differential equations: existence and uniqueness of solutions.

Week 4. Black-Scholes model and option pricing formula.

Week 5. Replication of contingent claims. European options.

Week 6. American options and their valuation.

Week 7. The PDE approach to hedging and pricing.

Week 8. Exotic (Asian, lookback, knock-out barrier,...) options.

Week 9. The role of the numeraire. Forward measure.

Week 10. Term-structure modelling: short rate models, affine models.

Week 11. Heath-Jarrow-Morton models. Defaultable bonds.

Week 12. Asset price models involving jumps.

Reference:

Steven E. Shreve: Stochastic calculus for finance, vols. I and II, Springer, 2004

QUANTITATIVE FINANCIAL RISK ANALYSIS

Course coordinator: Miklos Rasonyi

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Probability 1, Real Analysis

Course Level: advanced MS

Brief introduction to the course:

The main mathematical methods of financial risk analysis are presented like Credit portfolio risk models, or the Low default problem.

The goals of the course:

The main goal of the course is to introduce students to the main methods Qualitative Financial Risk Analysis.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Market risk measurement
2. Time independent fat tailed distributions of market price (FX rates, interest rates, stock and commodity prices) fluctuations
3. Volatility clusters in stock exchanges, GARCH models
4. Filtered historical simulation

5. Best practice for calculating Value at Risk for market risk related problems
6. Credit portfolio risk models
7. Mathematical background of the Basel II regulatory model
8. Granularity adjustment for undiversified idiosyncratic risk
9. CreditRiskPlus as a realistic and implementable portfolio model
10. Comparison of CreditRiskPlus and CreditMetrics models
11. Probability of Default (PD) Estimation
12. Low default problem

References:

1. R.N. Mantegna and H.E. Stanley: An Introduction to Econophysics, Correlations and Complexity in Finance, Cambridge University Press, 2000
2. M. K. Ong: Internal Credit Risk Models, Capital Allocation and Performance Measurement, Risk Books, 2000
3. Credit Suisse First Boston: CreditRisk+, A Credit Risk Management Framework, 1997

RISK MEASURES

Course coordinator: Miklos Rasonyi

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Probability 1, Real Analysis

Course Level: advanced MS

Brief introduction to the course:

The main mathematical methods of financial risk measures are presented.

The goals of the course:

The main goal of the course is to introduce students to the main properties of Risk Measures.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Quantiles. Value at Risk.
2. Utility indifference prices.
3. Coherent and convex risk measures. Examples.
4. Continuity properties of risk measures.
5. Dual representations.
6. Operations with risk measures (convolution, etc).
7. Law invariant risk measures.
8. The consequences of having "No Good Deals".
9. Conditional risk measures.
10. Time consistency.
11. Examples for dynamic risk measures.
12. Optimal risk-sharing between agents.

Reference:
Hans Follmer, Alexander Schied: Stochastic finance, Walter de Gruyter, 2011

BIOINFORMATICS

Course coordinator: Istvan Miklos

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: -

Course Level: introductory MS

Brief introduction to the course:

Stochastic models: HMMs, SCFGs and time-continuous Markov models and their algorithmic aspects.

The goals of the course:

The main goal of the course is to introduce students to the stochastic transformational grammars, especially HMMs and SCFGs, to time-continuous Markov models describing sequence evolution, and to the algorithmic background of these models.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

:
Lecture 1.
Theory: Score based dynamic programming algorithms. Linear, concave and affine gap penalties.

Lecture 2.
Theory: Conditional probability, Bayes theorem. Unbiased, consistent estimations. Statistical testing. Local alignment, extreme value distributions for local alignments, p and E value estimations.

Lecture 3.
Theory: Hidden Markov Models. Parsing algorithms: Forward, Backward and Viterbi. Posterior probabilities. Expectation Maximization. The Baum-Welch algorithm.

Lecture 4.
Theory: Profile HMMs. Aligning sequences via profile-HMMs. Pair-HMMs.
Practice: HMM topology design.

Lecture 5.
Theory: Substitution models. Felsenstein's algorithm for fast likelihood calculation of a tree.

Lecture 6.
Theory: Predicting protein secondary structures with profile HMMs and evolutionary models. Gene prediction with HMMs.

Lecture 7.
Theory: Modeling insertions and deletions with time-continuous Markov models: The Thorne-Kishino-Felsenstein models.

Lecture 8.
Theory: Describing the TKF models as pair-HMMs. Extension to many sequences: multiple-HMMs. The transducer theory for evolving sequences on an evolutionary tree.

Lecture 9.
Theory: Stochastic transformational grammars. Stochastic regular grammars are HMMs. Stochastic Context-Free Grammars. Parsing algorithms for SCFGs: Inside, Outside and CYK.

Lecture 10.
Theory: Posterior decoding of SCFGs. Expectation Maximization. Combining SCFGs with evolutionary models: the Knudsen-Hein algorithm.

Lecture 11.
Theory: Covarion Models as 'profile-SCFGs'. The RFam database. Predicting tRNAs in the human genome.

Lecture 12.
Theory: The Zuker-Tinoco model for RNA secondary structures. Calculating the partition function of the Boltzmann distribution and other moments of the Boltzmann distribution.

References:

1. Durbi-Eddy-Krogh-Mitchison: Biological sequence

2. http://www.renyi.hu/~miklosi/AlgorithmsOfBioinformatics.pdf

MATHEMATICAL MODELS IN BIOLOGY AND ECOLOGY

Course coordinator: IstvanMiklos

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Calculus, Ordinary Differential Equations

Course Level: introductory MS

Brief introduction to the course:

The main mathematical models in Biology and Ecology are discussed, like Predator-prey models, Reaction-diffusion equations and Evolutionary dynamics.

The goals of the course:

The main goal of the course is to introduce students to the Mathematical Models in Biology and Ecology.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Discrete and continuous single species models. Exponential and logistic growth. Thedelayed logistic equation

2. Multi-species communities: competition, comensualism, coexistence.

3. Predator-prey models. The Lotka-Volterra model and more complicated models (Gause,Kolmogorov). Prey-dependent and ratio-dependent predation.

4. Chemical reaction kynetics: Michaelis-Menten theory

5. The Kacser-Burns theory of steady-state systems.

6. Simple oscillatory reactions. Nerve impulses and Hodgkin-Huxley theory. FitzHugh-Nagumo model.

7. Reaction-diffusion equations. Convection, advection. Chemotaxis.

8. Evolutionary dynamics. Evolutionary Stable Strategies.

9. Basic concepts of mathematical epidemiology. Deterministic models. Compartmentalmodels. Single population models with constant population size. Models with no immunity.

10. Models with nonconstant population size and immunity effects. Basic reproduction numberof a disease. Stability and persistence.

11. Infective periods of fixed length. Models with delay. Arbitrarily distributed infectiveperiods. Seasonality and periodicity. Orbital stability of periodic solutions.

12. Numerical simulations and visualisations by means of XPP, Phaser, Maple (or equivalent).

References:

1. L. Edelstein-Keshet, Mathematical Models in Biology, SIAM Classics in AppliedMathematics 46, 2004

2. F. Brauer and C. Castillo-Chavez, Mathematical Models in Population Biology andEpidemiology, Springer, 2001

3. V. Capasso, Mathematical structures of Epidemic Systems, Lecture Notes inBiomathematics, Springer Verlag, Berlin 1993.

EVOLUTIONARY GAME THEORY AND POPULATION DYNAMICS

Course coordinator: IstvanMiklos

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Calculus, Ordinary Differential Equations

Course Level: introductory MS

Brief introduction to the course:

The main mathematical models in Biology and Ecology are discussed, like Predator-prey models, Reaction-diffusion equations and Evolutionary dynamics.

The goals of the course:

The main goal of the course is to introduce students to the Mathematical Models in Biology and Ecology.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Evolutionary stability. Normal form games. Evolutionarily stable strategies. Population games

2. Replicator dynamics. The equivalence of the replicator equation to the Lotka-Volterra equation. The rock-scissors-paper game. Partnetship games and gradients

3. Other game dynamics. Imitation dynamics. Monotone selection dynamics. Best-response dynamics. Adjustment dynamics. A universally cyclic game.

4. Adaptive dynamics. The repeated Prisoner's Dilemma. Adaptive dynamics and gradients.

5. Asymmetric games and replicator dynamics for them.

6. Bi-matrix games

7. Population dynamics and game dynamics

8. Game dynamics for Mendelian populations

9. Models in non-homogenous time and space.

10. Chaos in evolutionary games

11. Nesh equilibrium.

12. Numerical simulations and visualisations

Reference:

J. Hofbauer and K. Zigmund, Evolutionary Games and Population Dynamics, Cambridge University Press, 1998.

PROBABILISTIC MODELS OF THE BRAIN AND THE MIND

Course coordinator: Mate Lengyel

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Probability 1

Course Level: introductory MS

Brief introduction to the course:

The main probabilistic models of the Brain and the Mind are discussed, like neural representations of uncertainty, probabilistic neural networks and probabilistic population codes.

The goals of the course:

The main goal of the course is to introduce students to the Probabilistic Models of the Brain and the Mind.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Machine learning, unsupervised learning.
2. Bayesian networks, reinforcement learning, sampling algorithms.
3. Variational methods, computer vision.
4. Cognitive science, inductive reasoning.
5. Statistical learning, semantic memory,
6. Vision as analysis by synthesis.
7. Sensorimotor control, classical and instrumental conditioning.
8. Behavioural economics.

9. Neuroscience, neural representations of uncertainty.
10. Probabilistic neural networks.
11. Probabilistic population codes, natural scene statistics and efficient coding.
12. Neuroeconomics, neuromodulation.

References

1. Dayan & Abbott. Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems, MIT press, 2001.
2. MacKay. Information Theory, Inference & Learning Algorithms, Cambridge University Press, 2002.
3. Sutton &Barto. Reinforcement Learning: An Introduction, MIT Press, 1998.
4. Doya et al. Bayesian Brain: Probabilistic Approaches to Neural Coding, MIT Press, 2007.

5. Rao et al. Probabilistic Models of the Brain: Perception and Neural Function, MIT Press, 2002.

ERGODIC THEORY

Course coordinator: Peter Balint

No. of Credits: 3 and no. of ECTS credits: 6

Prerequisites: Real Analysis, Probability 1

Course Level: advanced MS

Brief introduction to the course:

Basic concepts of ergodic theory: measure preserving transformations, ergodic theorems, notions of ergodicity, mixing and methods for proving such properties, topological dynamics, hyperbolic phenomena, examples: eg. rotations, expanding interval maps, Bernoulli shifts, continuous automorphisms of the torus.

The goals of the course:

The main goal of the course is to give an introduction to the central ideas of ergodic theory, and to point out its relations to other fields of mathematics.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Basic definitions and examples(measure preserving transformations, examples: rotations, interval maps etc.)

Week 2: Ergodic theorems(Poincare recurrence theorem, von Neumann and Birkhoffergodic theorems)

Week 3: Ergodicity(different characterizations, examples: rotations)

Week 4: Further examples: stationary sequences(Bernoulli shifts, doubling map, baker's transformation)

Week 5: Mixing(different characterizations, study of examples from this point of view)

Week 6: Continuous automorphisms of the torus(definitions, proof of ergodicity via characters)

Week 7: Hopf's method for proving ergodicity(hyperbolicity of a continuous toralautomorphism, stable and unstable manifolds, Hopf chains)

Week 8: Invariant measures for continuous maps(Krylov-Bogoljubov theorem, ergodic decomposition, examples)

Week 9: Markov maps of the interval(definitions, existence and uniqueness of the absolutely continuous invariant measure)

Weeks 10-12: Further topics based on the interest of the students(eg. attractors, basic ideas of KAM theory, entropy, systems with singularities etc.)

References:

1. P. Walters:*Introduction to Ergodic Theory,* Springer, 2007

2. M. Brin- G.Stuck: *Introduction to Dynamical Systems,* Cambridge University Press 2002

MATHEMATICAL METHODS IN STATISTICAL PHYSICS

Course Coordinator: BalintToth

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Real Analysis, Probability 1

Course Level: advanced MS

Brief introduction to the course:

The main theorems of Statistical Physics are presented among others about Ising model.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Statistical Physics.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1 The object of study of statistical physics, basic notions.

Week 2-3 Curie-Weiss mean-field theory of the critical point. Anomalous fluctuations at the critical point.

Week 4-5 The Isingmodell on Zd.

Week 6-7 Analiticity I: Kirkwood-Salsburg equations.

Week 8-9 Analiticity II: Lee-Yang theory.

Week 10-11 Phase transition in the Ising model: Peierls' contour method.

Week 12 Models with continuous symmetry.

FRACTALS AND DYNAMICAL SYSTEMS

Course Coordinator: Karoly Simon

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Real Analysis, Probability 1

Course Level: advanced MS

Brief introduction to the course:

The main theorems about Fractals are presented among others about local dimension of invariant measures.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of the Fractals and Dynamical Systems.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1-2 Fractal dimensions. Hausdorff and Packing measures.

Week 3 Basic examples of dynamically defined fractals. Horseshoe, solenoid.

Week 4-5 Young's theorem about dimension of invariant measure of a C2 hyperbolic diffeomorphism of a surface.

Week 6-7 Some applications of Leddrapier- Young theorem.

Week 8-9 Barreira, Pesin, Schmeling Theorem about the local dimension of invariant measures.

Week 10-11 Geometric measure theoretic properties of SBR measure of some uniformly hyperbolic attractors.

Week 12 Solomyak Theorem about the absolute continuous infinite Bernoulli convolutions.

References:

1. K. Falconer, Fractal geometry. Mathematical foundations and applications. John Wiley & Sons, Ltd., Chichester, 1990.

2. K. Falconer, Techniques in fractal geometry. John Wiley & Sons, Ltd., Chichester, 1997.

3. Y. Pesin, Dimension theory in dynamical systems. Contemporary views and applications Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1997.

COMPUTATIONAL NUMBER THEORY

Course coordinator: Laszlo Csirmaz

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Algebra 1

Course Level: intermediate MS

Brief introduction to the course:

Number theory gained a huge popularity in the last decade as the freshly emerging methods in cryptography required generating really huge prime numbers. In this course we look at the methods the newest results in number theory which are easily available with a modest mathematical knowledge. There will be an overview of the heuristics applied routinely nowadays, as well as a detour into the more advanced topics. As the title suggest, the course requires some programming skills, but no actual programs will be written. Implementing some of the discussed algorithms and methods can be done voluntarily.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Computational Number Theory.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. Students learn several primality tests, factoring algorithms. Will have a basic knowledge about elliptic curves and methods using elliptic curves. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Prime numbers, prime formulas, special type of primes. Prime number theorem, analytic expressions.

2. Computing with large numbers: the grammar school method, multiplication,division and remainder, exponentiation

3. Montgomery method, binary and recursive gcd, Karatsuba and Tom-Cook methods, Fourier transformation. Schönhage method.

4. Chinese remainder theorem, applications, polynomial multiplication and polynomial evaluation

5. Recognizing primes: smooth numbers, sieving, Fermat and Carmichael numbers, Rabin-Muller test

6. Primality proving: Lucas-Lehmer test, divisors in residue classes, finite field primality test

7. Legendre and Jacoby symbol, square roots, quadratic forms, finding roots in a finite field, Polynomial algorithm for primality.

8. Exponential factoring algorithms: Fermat method, Lehman method, Pollard rho, Pollard lambda

9. Baby-step, giant-step algorithm, Pollard method, polynomial evaluation

10. Quadratic sieve, Number field sieve method

11. Elliptic curves, the Foldwasser-Kilianprimality test

12. The RSA cryptosystem, elliptic curve cryptosystem, El Gamal methods in cryptopgrahy.

Reference:

R. Crandall, C. Pomerance: Prime Numbers, Springer, 2001; A. Menezes, P. van Oorschot, and S. Vanstone: Handbook of Applied Cryptography, CRC Press, 1997

COMPUTATIONS IN ALGEBRA

Course coordinator: Pal Hegedus

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Algebra 1

Course Level: intermediate MS

Brief introduction to the course:

The course will cover many basic algorithms that are still used by computer algebra systems today. We shall cover mostly problems connected to abstract and linear algebra, but some outside areas will be touched especially if it mathematically related. The computer algebra system GAP will be used many times.

The goals of the course:

One of the main goals of the course is to introduce students to the most important concepts and fundamental results in computational algebra. A second goal is to make the student more comfortable with abstract algebra itself.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Introduction. Review of abstract algebraic notions.
2. Representation of finite fields. Applications.
3. Sims algorithm for finite permutation groups, appliactions.
4. Factorising polynomials over finite fields.
5. Factorisingpolynomials in several variables and over the rational field.
6. Algorithmic problems relating to lattices in the Euclidean space, the LLL algorithm.
7. Finding proime numbers.
8. Error correcting codes.
9. Ideal membership and Gröbner bases.

10-12. Practical sessions in the PhD study room will be evenly distributed among the weeks.

Optional topics:

Resultants, group representations, modules.

References:

1. J von zurGathen and J Gerhard, Modern Computer Algebra, Cambridge University Press, 1999.
2. Th Becker and V Weispfenning, Gröbner Bases: A Computational Approach to Commutative Algebra, Graduate Texts in Mathematics, Springer, 1998.

MATRIX COMPUTATIONS WITH APPLICATIONS

Course coordinator: Pal Hegedus

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: linear algebra

Course Level: introductory MS

Brief introduction to the course:

The course will cover most standard matrix manipulations. There will be ample examples and applications described.

The goals of the course:

The main goal of the course is to further strengthen students' understanding of linear algebra and that they understand ways of applying it to other areas of algebra and mathematics. They are expected to reach an ability of seeing the interdependencies among the various linear algebraic objects.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Introduction. Inverse, topological properties of the inverse, the Henderson-Searle and the Sherman-Morrison-Woodbury formulas. The Schur complement.
2. Gauss Elimination, elementary matrices, LU and LDU factorisation. Polynomials of matrices, the Cayley-Hamilton theorem.
3. Rank and index of matrices. Direct sum decomposition, one-sided inverses, Matrix equivalence, Full Rank Factorisation. Moore-Penrose inverse. Applications.
4. Other generalised inverses (1-inverse, reflexive, normal, Drazin).
5. Applications. Standard and more complicated matrix norms.
6. Orhogonality, symmetric and symplectic forms, Witt's theorems, Euclidean spaces of vacors/matrices, Gram-Schmidt orhogonalisation.
7. Orthogonal Full Rank Factorisation, QR decomposition and the least squares solution.
8. Diagonalisation, spectral questions, Jordan decomposition.
9. Jordan Normal Form. Schur decomposition, Singular Value Decomposition.
10. Error Correcting Codes. Hamming distance and weight. Bounds.
11. Hamming and Extended Hamming Codes. Binary Golay Code. Idea of BCH codes.
12. Cyclic Codes, Designed distance. Error locator polynomial, error evaluator polynomial, Berlekamp's algorithm.

Optional topics:

Linear groups, Hadamard matrices, ill-conditioned systems, algorithmic approaches.

References:

1. R Piziak, P L Odell, Matrix Theory (FromGeneralized Inverses to Jordan Form), Chapman & Hall/CRC, 2007.
2. S Barnett, Matrices (Methods and applications), Oxford University Press, New York, 1992.
3.Bierbrauer, Jurgen, Introduction to coding theory, Chapman & Hall/CRC, 2005.

CRYPTOGRAPHIC PROTOCOLS

Course coordinator: Laszlo Csirmaz

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Algebra 1, Introduction to Computer Science

Course Level: intermediate MS

Brief introduction to the course

Authentication and key establishment are fundamental building blocks for securing electronic communications. This course is an introduction to the study of protocols for authentication and key establishment as well as a general treatment of cryptographic protocols and the proof of their properties.

The goals of the course

To get acquainted with the notion of cryptographic protocols through several examples, including famous faulty protocols which fall short of some of their goals. Protocols for key establishment, key exchange, with symmetric and public key tools. Formalizing the goals of protocols and proving their correctness.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. What protocols are, examples, basic properties, adversaries against protocols.
2. Typical attacks, famous protocol errors. Protocols with more parties; simulability.
3. Zero Knowledge protocols, concurrent ZK, resettable ZK, non-interactive ZK protocols.
4. Protocols for authentication. Confidentiality, integrity, data origin authentication, non-repudiation.
5. Typical attacks on protocols: eavesdropping, modification, replay, men-in-the-middle, denial of service, typing attacks, examples.
6. Keys, key freshness, STS protocol, Key Transport Protocol.
7. Formal systems for protocol analysis: Murphi, BAN.
8. Using BAN to verify protocol properties. Protocols using trusted third party
9. Denning-Sacco, Otway-Rees, Kerberos protocols.
10. Protocols using public key infrastructure
11. Protocols using weak password schemes, the Katz-Ostrovsky-Yung protocol.
12. Attacks on the early WiFi protocols.

Reference:
Colin Boyd and AnishMaturia, Protocols for Authentication and Key Establishment, Springer, 2003

CRYPTOLOGY

Course coordinator: Laszlo Csirmaz

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: Basic Algebra 1, Introduction to Computer Science, Probability 1

Course Level: intermediate MS

Brief introduction to the course:

The main theorems and methods of Cryptology are presented like Public key cryptography, or Secret Sharing Schemes.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Cryptology.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Computational difficulty, computational indistinguishability
2. Pseudorandom function, pseudorandom permutation
3. Probabilistic machines, BPP
4. Hard problems
5. Public key cryptography
6. Protocols, ZK protocols, simulation
7. Unconditional Security,
8. Multiparty protocols
9. Broadcast and pairwise channels
10. Secret Sharing Schemes,
11. Verifiable SSS
12. Multiparty Computation

References:

1. Ivan Damgard (Ed), Lectures on Data Security, Springer 1999

2. OdedGoldreich, Modern Cryptography, Probabilistic Proofs and Pseudorandomness, Springer 1999

COMBINATORIAL OPTIMIZATION

Course coordinator: Ervin Gyori

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: basic linear algebra and combinatorics

Course Level: introductory MS

Brief introduction to the course:

Basic concepts and theorems are presented. Some significant applications are analyzed to illustrate the power and the use of combinatorial optimization. Special attention is paid to algorithmic questions.

The goals of the course:

One of the main goals of the course is to introduce students to the most important results of combinatorial optimization. A further goal is to discuss the applications of these results to particular problems, including problems involving applications in other areas of mathematics and practice. Finally, computer science related problems are to be considered too.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Typical optimization problems, complexity of problems, graphs and digraphs

Week 2: Connectivity in graphs and digraphs, spanning trees, cycles and cuts, Eulerian and Hamiltonian graphs

Week 3: Planarity and duality, linear programming, simplex method and new methods

Week 4: Shortest paths, Dijkstra method, negative cycles

Week 5: Flows in networks

Week 6: Matchings in bipartite graphs, matching algorithms

Week 7: Matchings in general graphs, Edmonds' algorithm

Week 8: Matroids, basic notions, system of axioms, special matroids

Week 9: Greedy algorithm, applications, matroid duality, versions of greedy algorithm

Week 10: Rank function, union of matroids, duality of matroids

Week 11: Intersection of matroids, algorithmic questions

Week 12: Graph theoretical applications: edge disjoint and covering spanning trees, directed cuts

Reference:
E.L. Lawler, Combinatorial Optimization: Networks and Matroids, Courier Dover Publications, 2001 or earlier edition: Rinehart and Winston, 1976

NONLINEAR OPTIMIZATION

Course coordinator: Sandor Bozoki

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: basic linear algebra

Course Level: introductory MS

Brief introduction to the course:

The course provides an introduction to the nonlinear optimization problems. Main topics are the first- and second-order, necessary and sufficient optimality conditions; convex optimization; quasiconvex and pseudoconvex functions; Lagrange duality, weak and strong duality theorems, saddle point theorem; Newton's method in optimization, theorems of convergence.

The goals of the course:

The aim of the course is to encourage students to the use of nonlinear optimization techniques in many areas of their interest and to gain theoretical and practical knowledge. Students are proposed to know the elementary theorems and proofs of nonlinear optimization and also to use the corresponding tools and commands in Matlab and/or Maple.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems.Students can identify, model and classify nonlinear optimization problems and can solve some of them by using Lagrange multipliers or Newton's method. Students will have a toolbox of basic nonlinear optimization routines as well as the ability of implementing elementary algorithms. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Modeling of nonlinear optimization problems – examples, well known mathematical problems written as nonlinear optimization problems, alternative ways for modeling the same problem

Week 2-3: First- and second-order, necessary and sufficient optimality conditions - and solution of numerical exercises

Week 3: Convex optimization – theorems of convex optimization, applications in inequalities

Week 5: An introduction to the generalized convexity: quasiconvex and pseudoconvex functions – with examples and counterexamples

Week 6: Lagrange duality – relation to the primal problem, solution of numerical exercises

Week 7: Duality theorems

Week 8: Saddle point theorem

Week 9: Newton's method in optimization, theorems of convergence

Week 10-11: The implementation of Newton's method in one and two dimensions – in Matlab and/or Maple

Week 12: Newton's method and fractals

References:
1. TamásRapcsák, Smooth Nonlinear Optimization in Rn, Kluwer Academic Publishers, 1997.

2. Pascal Sebah, Xavier Gourdon: Newton's method and high order iterations

3. http://numbers.computation.free.fr/Constants/Algorithms/newton.html

4. http://numbers.computation.free.fr/Constants/Algorithms/newton.ps

TOPICS IN NONLINEAR OPTIMIZATION

Course coordinator: Sandor Bozoki

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: basic linear algebra

Course Level: Intermediate MS

Brief introduction to the course:

The course discusses advanced nonlinear optimization problems.

The goals of the course:

The aim of the course is to enable students to use advanced nonlinear optimization techniques in many areas of mathematics and practical applications, including mastering the corresponding tools and commands in Matlab and/or Maple.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. They develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Problems in combinatorics, number theory, and geometry as nonlinear optimization problems
2-3. Multivariate polynomial systems and their applications
4. Homotopy continuation method
5. Verification of the roots
6. Fractional programming
7. Sensitivity analysis in multi-attribute decision models
8-9. Eigenvalue minimization problems, application to incomplete pairwise comparison matrices in multi-attribute decision making
10. Pareto optimality
11-12. Nonlinear optimization in Matlab

References:

Boyd, Vandenberghe: Convex Optimization. Cambridge University Press, 2009

Bozóki, S., Lee, T.L., Rónyai, L. (2015): Seven mutually touching infinite cylinders, Computational Geometry: Theory and Applications, 48(2), pp.87-93. DOI 10.1016/j.comgeo.2014.08.007

Bozóki, S., Fülöp, J., Rónyai, L. (2010): On optimal completions of incomplete pairwise comparison matrices, Mathematical and Computer Modelling, 52(1-2), pp.318-333. DOI 10.1016/j.mcm.2010.02.047

Firsching (2016): Optimization methods in discrete geometry, PhD dissertation, Freie Universität Berlin.

Mészáros, Cs., Rapcsák, T. (1996): On sensitivity analysis for a class of decision systems. Decision Support Systems, 16(3), pp.231-240. DOI 10.1016/0167-9236(95)00012-7

OPTIMIZATION IN ECONOMICS

Course coordinator: Sandor Bozoki

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: basic linear algebra

Course Level: introductory MS

Brief introduction to the course:

In the last decades mathematical methods have become indispensable in the study of many economical problems, in particular, in the optimization of certain real-life phenomena. For instance, J. F. Nash received the Nobel Prize in Economics (1994) for his outstanding contributions in the field of Economicsvia mathematical tools. Our aim here is to emphasize the importance of Mathematics in the study of a broad range of economical problems. Many applications/examples will be discussed in detail.

The goals of the course:

The main goal of the present course is to introduce Students into the most important concepts and fundamental results of Economics by using various tools from Mathematics as calculus of variations, critical points, matrix-algebra, or even Riemannian-Finsler geometry. Starting with basic economical problems, our final purpose is to describe some recent research directions concerning certain optimization problems in Economics.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Lecture1. Introduction and motivation: some basic problems from Economics via optimization.

Lecture 2. Economic applications of one-variable calculus (demand and marginal revenue, elasticity of price, cost functions, profit-maximizing output).

Lecture 3. Economic applications of multivariate calculus (consumer choice theory, production theory, the equation of exchange in Macroeconomics, Pareto-efficiency, application of the least square method).

Lectures4.Linear programming (application of the geometric, simplex and dual simplex method).

Lecture 5.Linear economical problems (diet problem, Ricardian model of international trade).

Lecture 6. Comparative statics I (equilibrium comparative statics in one and two dimensions; comparative statics with optimization, perfectly competitive firms, Cournot duopoly model).

Lecture 7. Comparative statics II: n variables with and without optimization (equilibrium comparative statics in n dimensions, Gross-substitute system, perfectly competitive firms).

Lecture 8. Comparative statics III: Optimization under constraints (Lagrange-multipliers, specific utility functions, expenditure minimization problems).

Lecture 10. Nash equilibrium points (existence, location, dynamics, and stability).

Lecture 11. Optimal placement of a deposit between markets: a Riemann-Finsler geometrical approach.

Lecture 12.Economical problems via best approximations.

References:

1. J.-P.Aubin, Optima and Equilibria, An Introductionto Nonlinear Analysis, Springer-Verlag, Berlin, Heidelberg, 1993.
2. J.-P.Aubin, Analyse non lineaire et ses motivationseconomiques, Masson, 1984.
3. D. W. Hands, Introductory Mathematical Economics, D.C. Heath and Company, Toronto, 1991.
4. I.V. Konnov, Equilibrium models and Variational Inequalities, Math. in Science and Engineering, Elsevier, Amsterdam, 2007.
5. R. Wild, Essential of Production and Operations Management, Cassel, London, 1995.

INTRODUCTION TO DISCRETE MATHEMATICS

Course coordinator: Ervin Gyori

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: -

Course Level: introductory MS

Brief introduction to the course:

Fundamental concepts and results of combinatorics and graph theory. Main topics: counting, recurrences, generating functions, sieve formula, pigeonhole principle, Ramsey theory, graphs, flows, trees, colorings.

The goals of the course:

The main goal is to study the basic methods of discrete mathematics via a lot of problems, to learn combinatorial approach of problems. Problem solving is more important than in other courses!

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1. Basic counting problems, permutations, combinations, sum rule, product rule
Week 2. Occupancy problems, partitions of integers

Week 3. Solving recurrences, Fibonacci numbers

Week 4. Generating functions, applications to recurrences

Week 5. Exponential generating functions, Stirling numbers, derangements

Week 6. Advanced applications of generating functions (Catalan numbers, odd partitions)

Week 7. Principle of inclusion and exclusion (sieve formula), Euler function
Application of sieve formula to Stirling numbers, derangements, and other involved problems

Week 8. Pigeonhole principle, Ramsey theory, ErdosSzekeres theorem

Week 9. Basic definitions of graph theory, trees

Week 10. Special properties of trees, Cayley's theorem on the number of labeled trees

Week 11. Flows in networks, connectivity

Week 12. Graph colorings, Brooks theorem, colorings of planar graphs

References:

1. Fred. S. Roberts, Applied Combinatorics, Prentice Hall, 1984
2. Fred. S. Roberts, Barry Tesman, Applied Combinatorics, Prentice Hall, 2004
3. BelaBollobas, Modern Graph Theory, Springer, 1998

GRAPH THEORY AND APPLICATIONS

Course coordinator: Ervin Gyori

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: -

Course Level: introductory MS

Brief introduction to the course:

In recent years in the study of networks (web, VLSI, etc.) graph theory became central in applications of mathematical methods to everyday problems. The course is to review the most important questions related to graphs emphasizing on subjects with practical applications as well as applications in other areas of mathematics. Furthermore, we are going to deal with the algorithmic aspects, though we are not to cover all details of implementation, etc.

The goals of the course:

The main goal of the course is to introduce students to some important graph theoretical methods and to show their applicability to various problems. We intend to discuss graph algorithms as well as theoretical results.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1: Basic concepts

Week 2: Euler trails, Hamilton cycles, sufficient conditions

Week 3: Disjoint cycles, 2-factors

Week 4: Chromatic number of graphs, Brooks' theorem, other estimates

Week 5: Edge colorings, list chromatic number, and other coloring parameters

Week 6: Matchings in bipartite graphs, matchings in arbitrary graphs, Tutte's theorem, matching algorithms

Week 7: Flows in networks, applications, Menger's theorems

Week 8: Highly connected graphs, Gyori-Lovasz theorem, linkages

Week 9: Planar graphs, Kuratowski theorem, colorings of maps and planar graphs

Week 10: Extremal graphs, Turan theorem, Ramsey theorem and applications

Week 11: Probabilistic proofs, linear algebraic proofs

Week 12: Graph algorithms, minimum cost spanning trees, DFS and BFS spanning trees and their applications

Reference: R. Diestel, Graph Theory, Springer, 2005+ handouts

PACKING AND COVERING

Course Coordinator: KarolyBoroczky

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: calculus

Course Level: introductory MS

Brief introduction to the course:

The main theorems of Packings and Coverings by convex bodies in the Euclidean space, and by balls in the spherical and hyperbolic spaces concentrating on density.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of the Theory of Packing and Covering.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Theorem of Groemer concerning the existence of densest packings and thinnest coverings. Dirichlet cells, Delone triangles.
2. Theorems of Thue and Kershner concerning densest circle packings and thinnest circle coverings. Packing and covering of incongruent circles. Theorems of Dowker, generalized Dirichlet cells. Packing and covering of congruent convex discs: theorems of C.A. Rogers and L.FejesTóth.
3. The moment theorem. Isoperimetric problems for packings and coverings. Existence of dense packings and thin coverings in the plane: p-hexagons, extensive parallelograms, theorems of W. Kuperberg, D. Ismailescu, G. Kuperberg and W. Kuperberg. The theorem of E. Sas.
4. Multiple packing and covering of circles.
5. The problem of Thammes; packing and covering of caps on the 2-sphere. The moment theorem on S2, volume estimates for polytopes containing the unit ball.
6. Theorem of Lindelöff, isoperimetric problem for polytopes. Packing and covering in the hyperbolic plane.
7. Packing of balls in Ed the method of Blichfeldt, Rogers' simplex bound. Covering with balls in Ed the simplex bound of Coxeter, Few and Rogers.
8. Packing in Sd, the linear programming bound. Theorem of Kabatjanskii and Levenstein.
9. Existence of dense lattice packings of symmetric convex bodies: the theorem of Minkowski-Hlawka.
10. Packing of convex bodies, difference body, the theorem of Rogers and Shephard concerning the volume of the difference body.
11. Construction of dense packings via codes.
12. The theorem of Rogers concerning the existence of thin coverings with convex bodies. Approximation of convex bodies by generalized cylinders, existence of thin lattice coverings with convex bodies

References:

1. L. FejesTóth: Regular figures, Pergamon Press, 1964.

2. J. Pach and P.K. Agarwal: Combinatorial geometry, Academic Press, 1995.

3. C.A. Rogers: Packing and covering, Cambridge University Press, 1964.

CONVEX POLYTOPES

Course Coordinator: KarolyBoroczky

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: basic linear algebra

Course Level: introductory MS

Brief introduction to the course:

The main theorems about the combinatorial structure of convex polytopes are presented concentrating on the numbers of faces in various dimensions.

The goals of the course:

To introduce the basic combinatorial properties of convex polytopes.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Polytopes as convex hull of finite point sets or intersections of halfspaces.
2. Faces of polytopes.
3. Examples: Simplicial, simple, cyclic and neighbourlypolytopes.
4. Polarity for polytopes.
5. The Balinski theorem.

6. Discussion of the Steinitz theorem for three polytopes.
7. Realizability using rational coordinates.
8. Gale transform and polytopes with few vertices.
9. The oriented matroid of a polytope
10. Shelling.Euler-Poincaré formula
11. h-vector of a simplicialpolytope, Dehn-Sommerfield equations
12. Upper bound theorem Stresses Lower bound theorem Weight algebra Sketch of the proof of the g-theorem.

Reference: G.M. Ziegler: Lectures on polytopes. Springer, 1995.

COMBINATORIAL GEOMETRY

Course coordinator: KarolyBoroczky

No. of Credits: 3 and no. of ECTS credits 6

Prerequisites: -

Course Level: introductory MS

Brief introduction to the course:

Convexity, separation, Helly, Radon, Ham-sandwich theorems, Erdős-Szekeres theorem and its relatives, incidence problems, the crossing number of graphs, intersection patterns of convex sets, Caratheodory and Tverberg theorems, order types, Same Type Lemma, the k-set problem

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Combinatorial Geometry.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

week 1: convexity, linear and affine subspaces, separation

week 2: Radon' theorem, Helly's theorem, Ham-sandwich theorem

week 3: Erdos-Szekeres theorem, upper and lower bounds

week 4: Erdos-Szekeres-type theorems, Horton sets

week 5: Incidence problems

week 6: crossing numbers of graphs

week 7: Intersection patterns of convex sets, fractional Helly theorem, Caratheodory theorem

week 8: Tverberg theorem, order types, Same Type Lemma

week 9-10: The k-set problem, duality, k-level problem, upper and lower bounds

week 11-12: further topics, according to the interest of the students

Reference: J. Matousek: Lectures on Discrete Geometry, Springer, 200

GEOMETRY OF NUMBERS

Course Coordinator: KarolyBoroczky

No. of Credits: 3, and no. of ECTS credits: 6

Prerequisites: linear algebra, calculus

Course Level: introductory MS

Brief introduction to the course:

The main theorems of Geometry of Numbers are presented, among others the Minkowski theorems, basis reduction, and applications to Diophantine approximation.

The goals of the course:

The main goal of the course is to introduce students to the main topics and methods of Geometry of Numbers.

The learning outcomes of the course:

By the end of the course, students are enabled to do independent study and research in fields touching on the topics of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Lattices, sublattices, bases, determinant of a lattice.
2. Convex bodies, elements of the Brunn-Minkowski theory, duality, star bodies. Selection theorems of Blaschke and Mahler.
3. The fundamental theorem of Minkowski, and its generalizations: theorems of Blichfeldt, van der Corput.
4. Successive minima, Minkowski's second theorem.
5. The Minkowski-Hlawka theorem.
6. Reduction theory, Korkine-Zolotarev basis, LLL basis reduction.
7. Connections to the theory of packings and coverings.
8. Diophantine approximation: simultaneous, homogeneous, and inhomogeneous.
9. Theorems of Dirichlet, Kronecker, Hermite, Khintchin
10. Short vector problem, nearest lattice point problem Applications in combinatorial optimization.
11. The flatness theorem.
12. Covering minima Algorithmic questions, convex lattice polytopes.

References:

1. J.W.S Cassels: An introduction to the geometry of numbers, Springer, Berlin, 1972.

2. P.M. Gruber, C.G. Lekkerkerker: Geometry of numbers, North-Holland, 1987.

3. L. Lovász: An algorithmic theory of numbers, graphs, and convexity, CBMS-NSF regional conference series, 1986.

EXACT AND APPROXIMATE ALGORITHMS FOR VEHICLE ROUTING PROBLEMS

Course coordinator: Istvan Miklos

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: Theory of Algorithms

Course Level: Intermediate MS

Brief introduction to the course:

The vehicle routing problem is to find an optimal schedule to collect and/or deliver objects with a fleet of vehicles. The course gives an introduction to this topic, starting with the mathematical formulization of the problem and then introducing the most important exact and approximate solutions.

The goals of the course:

To give an in depth introduction to vehicle routing algorithms, discussing for example exact algorithms and tabu search algorithms, and to provide exact and approximate solutions.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. The Vechile Routing Problem: formulation, naive algorithms.
   Variants by adding different constraints.
   2. The Clarke and Wright algorithm.
   3. The sweeping algorithm (Wren and Alan)
   4. Exact algorithms I.
   5. Exact algorithms II.
   6. The Christofides-Mingozzi-Toth two-phase algorithm I.
   7. The Christofides-Mingozzi-Toth two-phase algorithm II.
   8. A tabu search algorithm (Gendreau, Hertz and Laporte, 1991) I.
   9. A tabu search algorithm (Gendreau, Hertz and Laporte, 1991) II.
   10. Simulated Annealing methods
   11. Parallel Tempering methods
   12. Further heuristic algorithms

Reference:

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.89.3073&rep=rep1&type=pdf
(Laporte: The Vehicle Routing Problem: An overview of exact and approximate algorithms, European Journal of Operational Research 59 (1992) 345-358)

MATHEMATICAL FINANCE: A PRIMER FOR PRACTITIONERS

Course coordinator: Tamas Matrai (MSCI)

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: -

Course Level: Intermediate MS

Brief introduction to the course:

The course is an introduction into tools to measure risk in practice

The goals of the course:

To give an in depth introduction to methods and theory used in practice

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

Week 1 Risk and return: risk measures, risk premium, evaluation of investment opportunities, economic capital
Week 2 Asset pricing: investor preferences, utilities, risk aversion, market/credit/macroeconomic risk, long-run risk
Week 3 Asset allocation: portfolio optimization (from Markowitz to Black-Litterman), managing to risk
Week 4-5 Investors and investment vehicles: from seeking alpha to index tracking (hedge funds, active asset managers, ETFs, etc.)
Week6 Model validation: model risk, impact of estimation noise, parameterization validation, backtesting
Week 7-8 Regulation: banking regulation (Basel), insurance regulation (Solvency), impact of regulation on finance industry, stress testing

Week 9-10 Credit & Liquidity: portfolio credit, counterparty credit, liquidity (theory & practice)
Week 11-12 Market microstructure: market models, trading platforms, traders

SCIENTIFIC PYTHON

Course coordinator: Roberta Sinatra

No. of Credits: 3, and no. of ECTS credits: 6

Time Period of the course: Fall Semester

Prerequisites: Basic programming skills in any programming language (e.g. familiarity with logical statements, for loops, with different variables), Basic statistics

Course Level: Intermediate MS

Brief introduction to the course: This course will provide a comprehensive, fast-paced introduction to Scientific Python. The course will run with theoretical classes, hands-on sessions and tutorials. We expect you to come to lectures and labs, ask questions when you get stuck, and develop a project taking advantage of tutorials.

The goals of the course:
The overarching goal is to equip students with enough programming experience to start working in any area of computation and data-intensive research. This course will lay a foundation from which new tools and techniques can be explored.

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

Tentative calendar
First week
1st class: introduction to Python and IPython notebook
2nd class: running a script, type of variables, data structures (lists, tuples, dictionaries)
3rd class: hands on session on lists, an example of web scraping.

Second week
4th class: Functions and packages. File parsing.
5th class: Hands-on session on dictionaries and file parsing

6th class: Visualization and Matplotlib

Third week
7th class: Characteristics of Numpy. Data analysis with Numpy
8th class: Hands-on session on networks (Networkx)
9th class: Scipy, Scikit-learn and machine learning applications

Fourth week
10th class: Pandas, data-frames

Reference

- Bill Mark Lutz, Learning Python, O'Reilly (2013) – Also available for free online
- Bill Lubanovic, Introducing Python, O'Reilly (2014)
- Wes McKinney, Python for Data Analysis, O'Reilly (2013)
- Online resources and documentation provided during classes

HOW TO GAMBLE IF YOU MUST

Course coordinator: Istvan Berkes

No. of Credits: 3, and no. of ECTS credits: s6

Time Period of the course: Fall Semester

Prerequisites: Basic principles of probability

Course Level: Introductory MS

Brief introduction to the course:

How to play roulette to maximize your chances to reach a previously determined target sum? If you interview 20 applicants for a job, then how to maximize your chances to choose the best candidate if after each interview you must make a decision on hiring or rejecting the applicant and previously rejected candidates cannot be called back. If you perform an experiment with two outcomes (success/failure) repeatedly, then when to stop to maximize the expected ratio of successes. If you have the option to buy a stock within 3 months at a fixed price K, then when to exercise the option to maximize your profit. These are mathematical questions with great practical importance and in this course we develop a theory for their solution.

The goals of the course:

To give an in depth introduction to understand how stochastic is used to find optimal strategies

The learning outcomes of the course:

By the end of the course, students are experts on the topic of the course, and how to use these methods to solve specific problems. In addition, they develop some special expertise in the topics covered, which they can use efficiently in other mathematical fields, and in applications, as well. They also learn how the topic of the course is interconnected to various other fields in mathematics, and in science, in general.

More detailed display of contents (week-by-week):

1. Optimal strategies in unfavorable games
Week 1 Bold play: theorem of Dubins and Savage
Week 2 Excessive functions, classification of casinos, the casino inequality
Week 3 Optimal strategy of roulette; Smith's theorem
Week 4 Analysis of some special games
2. Favorable games: How to win if you can
Week 5 Breiman's theorem on optimal strategies
Week 6 Multi- game casinos
Week 7 Kelly's betting formula
Week 8 Favorable games and Shannon's entropy
3. Optimal stopping
Week 9 Basic principles, backward induction
Week 10The marriage problem and the secretary problem
Week 11 Optimal stopping for the head and tail game - Optimal stopping of Markov chains
Week 12 Stock options: European, American and Asian

Assessment

To get credit, a 2-3 pages long case study should be written on a topic chosen by the student.