Differential Equations Workshop

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ABSTRACTS

Nicuşor Costea ("Simion Stoilow" Institute of Mathematics of the Romanian Academy and Politehnica University of Bucharest)

Producing bounded Palais-Smale sequences for locally Lipschitz functionals and applications to PDE's

At the beginning of 1990's, Schechter developed a theory for finding bounded Palais-Smale sequences for C^1-functionals defined on a Hilbert space, by proving a deformation lemma which did not require the Palais-Smale compactness condition, but using instead the restriction of the function to a closed ball of radius R and imposing a boundary condition on a region of the corresponding sphere which prevents deformations from exiting the ball. Dropping the boundary condition and imposing a mild compactness condition one obtains either a critical point or an eigenvalue.

A natural question arises:

Can this be done for functionals, not necessarily differentiable, defined on a Banach space?

We provide a positive answer assuming that the functional is locally Lipschitz and the space is reflexive and has strictly convex dual.

As applications we consider Dirichlet partial differential inclusions of the type

$$\begin{cases} -Au \in \partial_C f(x,u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

with Au being either the p-Laplacian ($\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$), or the Φ-Laplacian ($\Delta_\Phi u = \operatorname{div}(\frac{\varphi(|\nabla u|)}{|\nabla u|}\nabla u)$, $\Phi(t) = \int_0^t \varphi(s)\, ds$) and the nonlinearity f is measurable w.r.t. the first variable and locally Lipschitz w.r.t. the second variable. Remarkable differences occur due to the loss of homogeneity of the differential operator.

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Sara Daneri (FAU Erlangen-Nürnberg)

The Cauchy problem for dissipative solutions of the Euler equations up to Onsager's critical exponent

Our work is related to Onsager's conjecture, proven by Isett and refined by Buckmaster, De Lellis, Székelyhidi and Vicol, according to which below Hölder regularity $1/3$ there exist solutions of the incompressible Euler equations which dissipate the total kinetic energy. We deal with the Cauchy problem for such kind of solutions. We improve a joint work with L. Szekelyhidi, where we proved the existence of infinitely many Hölder $1/5-\varepsilon$ initial data, each one admitting infinitely many Hölder $1/5-\varepsilon$ solutions with pre-assigned total kinetic energy, raising the exponent of this wild initial data and solutions to the optimal $1/3-\varepsilon$. This is a joint work with E. Runa and L. Székelyhidi.

Eduard Feireisl (Institute of Mathematics, Czech Academy of Sciences)

On weak (measure-valued) solution approach to problem in fluid mechanics

We introduce the concept of dissipative measure-valued solutions to certain problems in fluid mechanics, in particular the complete Navier-Stokes-Fourier system and the complete Euler system. We discuss stability of strong solutions in this class (weak–strong uniqueness) and related questions. We also show the existence of these solutions that maximize the entropy production rate which may be seen as a kind of useful selection criterion.

István Faragó (ELTE and TU Budapest)

Qualitatively reliable numerical models of time-dependent problems with applications

In the modeling process we construct mathematical and numerical models. Both models should preserve the basic (physically motivated) qualitative properties of the original phenomena. In this talk this problem will be discussed. We examine the different qualitative properties (maximum principles, non-negativity preservation, maximum norm contractivity) for both models and we show the relation between them for the linear problems. For the numerical models we give the condition for the construction of the mesh under which the above qualitative properties are valid. We show that the condition of the convergence for the numerical models is typically weaker than the above conditions. The results will be demonstrated in different real-life problems. First, we formulate these conditions for the heat conduction problem. Then, the compartmental epidemic models which take into the account the space distribution will be considered For this problem we construct different discrete (finite difference) models and for fixed space partition we give conditions for the time-stepping parameter under which the main qualitative properties are preserved. We examine the sharpness of the given conditions, too.

Paul Georgescu (Technical University of Iaşi)

The global dynamics of a HIV transmission model with high risk groups

Joint with Y.-H. Hsieh (China Medical University, Taiwan) and C.J. Sun (Kunming University of Science and Technology, P.R. China)

We propose a compartmental model for HIV transmission with two high risk groups, female sex workers (SWs) and male injecting drug users (IDUs), along with a bridge group of male drug-free clients (DFCs). Two transmission routes are accounted for: needle sharing between IDUs and commercial sex between SWs and IDUs or DFCs, two compartments being considered for each group depending on disease stage.

To establish global stability properties, we use the graph theoretic approach of Guo, Li and Shuai for an abstract disease propagation model introduced *ad hoc* which features a product incidence given in a generic, unspecified form. We then establish the stability properties of both the disease-free equilibrium and the endemic equilibrium in terms of a basic reproduction number, which is seen to be a threshold parameter as far as the stability of the system is concerned. The global stability of the endemic equilibrium is obtained in terms of sign conditions which are a priori satisfied for a large class of functions which are suitable to represent forces of infection. Stability results for the originating HIV transmission model are then obtained via suitable particularizations, possible extensions of this model being also outlined.

To establish mitigation and eradication strategies for the spread of the disease, we obtained partial reproduction numbers for each disease transmission route in the model, explicit conditions for the global stability of equilibria with immediate practical significance being then derived in terms of the partial reproduction numbers.

Tihomir Gyulov (University of Ruse Angel Kanchev)

Existence of solutions to a model for option valuation in a market with switching liquidity

We consider a model for European option valuation in a market switching between liquid and illiquid state. It extends previous work of Ludkovski and Shen as soon as it includes multiple assets. We study the existence of solutions of the corresponding system of partial/ordinary differential equations.

Alexandru Kristaly (University Babes-Bolyai)

Sobolev inequalities on curved spaces: sharpness, volume non-collapsing and rigidities

It is well known that the curvature influences the validity of sharp Sobolev inequalities on curved spaces. In this talk we prove that a metric measure space curved in the sense of Lott-Sturm-Villani and supporting a Sobolev-type inequality, has a non-collapsing volume growth. Due to the quantitative character of the volume growth estimate, we establish several rigidity results on Riemannian manifolds with non-negative Ricci curvature supporting Sobolev-type inequalities by exploring a quantitative Perelman-type homotopy construction.

Mihai Mihăilescu(Department of Mathematics, University of Craiova and "Simion Stoilow" Institute of Mathematics of the Romanian Academy)

Problems Involving Rapidly Growing Operators in Divergence Form

Joint work with Marian Bocea

First, the family of partial differential equations $-\varepsilon\Delta u - 2\Delta_\infty u = 0$ ($\varepsilon > 0$) is studied in a bounded domain Ω for given boundary data. In the case where $\varepsilon = 1$, which is closely related to the study of exponentially harmonic maps, we establish existence and uniqueness of a classical solution as the unique minimizer in a closed subset of an Orlicz-Sobolev space of the appropriate energy functional associated to this problem - the integral over Ω of the exponential energy density $u \mapsto \frac{1}{2}\exp(|\nabla u|^2)$. We also explore the connections between the classical solutions of these problems and infinity harmonic and harmonic maps by studying the limiting behavior of the solutions as $\varepsilon \to 0^+$ and $\varepsilon \to \infty$, respectively. In the former case, we recover a result of L. C. Evans & Y. Yu (CPDE, 2007).

Next, the minimization problem

$$\Lambda_1(p) := \inf_{u\in X_0\setminus\{0\}} \frac{\displaystyle\int_\Omega (\exp(|\nabla u|^p) - 1)\, dx}{\displaystyle\int_\Omega (\exp(|u|^p) - 1)\, dx},$$

where $X_0 = W^{1,\infty}(\Omega)\cap(\cap_{q>1} W_0^{1,q}(\Omega))$, is studied when $\Omega \subset \mathbb{R}^D$ ($D \geq 1$) is an open, bounded, convex domain with smooth boundary and $p \in (1,\infty)$. We show that $\Lambda_1(p)$ is either zero, when the maximum of the distance function to the boundary of Ω is greater than 1, or it is a positive real number, when the maximum of the distance function to the boundary of Ω belongs to the interval $(0,1]$. In the latter case we provide estimates for $\Lambda_1(p)$ and show that for $p \in (1,\infty)$ sufficiently large $\Lambda_1(p)$ coincides with the principal frequency of the p-Laplacian in Ω. Some particular cases and related problems are also discussed.

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Gheorghe Moroşanu (CEU)

Approximate solutions to the telegraph differential system

Consider in $D = \{(x,t);\, 0 < x < 1,\, 0 < t < T\}$, the telegraph differential system

$$\{\, L\, u_t - v_x + Ru = f_1(x,t)\,, Cv_t - u_x + Gv = f_2(x,t) \qquad (1)$$

(see, for example, K. L. Cooke and D. W. Krumme, Differential-difference equations and nonlinear initial boundary-value problems for linear hyperbolic partial differential equations, *J. Math. Anal. Appl.*, **24** (1968), 372–387.). In practice $0 < L =$ inductance; $0 \le R =$ resistance; $0 < C =$ capacitance per unit length; $0 \le G =$ conductance; $f_1(x,t) =$ voltage per unit length impressed along the line in series with it; $f_2(x,t) = 0$; $u = u(t,x) =$ current flowing in the line; $v = v(t,x) =$voltage across the line.

We associate with (1) some boundary conditions, for example,

$$v(0,t) = ru(0,t),\ -u(1,t) = h(v(1,t)),\, 0 < t < T, \qquad (2)$$

and initial conditions

$$u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ 0 < x < 1\,. \qquad (3)$$

Here h is a continuous nondecreasing function. If h is linear then both equations in (2) express Ohm's law. R, G and r are either constants or nonlinear functions.

For existence and uniqueness of the solution (u,v) to problem (1), (2), (3) see, for example, Chapter III, G. Moroşanu, *Nonlinear Evolution Equations and Applications*, D. Reidel, Dordrecht–Boston–Lancaster–Tokyo, 1988.. Following an idea of J. L. Lions, we construct regularizations of problem (1), (2), (3) by adding the terms $-\varepsilon u_{tt}$ and $-\varepsilon v_{tt}$, $\varepsilon > 0$, to the left-hand sides of the two equations in (1), plus some conditions at $t = T$ for u and v (more precisely, either $u(\cdot,T) = u_T,\ \ v(\cdot,T) = v_T$ or $u_t(\cdot,T) = 0,\ \ v_t(\cdot,T) = 0$) in order to obtain complete problems. The solutions of these new problems are **more regular** (with respect to t) than (u,v), and for ε sufficiently small they approximate (u,v) (see L. Barbu and G. Moroşanu, Elliptic-like regularization of a fully nonlinear evolution inclusion and applications, *Comm. Contemp. Math.*, **19** (2017), No. 5, 1650037, 16 pp. and L. Barbu and G. Moroşanu, Elliptic-like regularization of semilinear evolution equations and applications to some hyperbolic problems, *J. Math. Anal. Appl.*, **449** (2017), No. 2, 966-978.).

On the other hand, if the inductance L occuring in (1) is small enough and R is a positive constant, then (u, v) is close to the **more regular** solution of the parabolic problem obtained by setting $L = 0$ in (1) and removing the condition $u(x, 0) = u_0(x), \quad 0 < x < 1$ (cf L. Barbu and G. Moroşanu, *Singularly Perturbed Boundary-Value Problems*, Birkhäuser, Basel–Boston–Berlin, 2007.) .

Miklós Rásonyi (CEU and Renyi Institute)

On fixed gain recursive estimators with discontinuity in the parameters

Joint with N. H. Chau, Ch. Kumar and S. Sabanis

In this talk we report progress on the convergence theory of stochastic gradient methods. We estimate the tracking error of a fixed gain stochastic approximation scheme. The underlying process is not assumed Markovian, a mixing condition is required instead. Furthermore, the updating function may be discontinuous in the parameter.

Elisabetta Rocca (University of Pavia)

Dissipative solutions for a hyperbolic system arising in liquid crystals modeling

In this talk we present the results contained in the paper (E. Feireisl, E. Rocca, G. Schimperna, A. Zarnescu, On a hyperbolic system arising in liquid crystals modeling, preprint arXiv:1610.07828, 1–22, Journal of Hyperbolic Differential Equations, to appear (2018)).

We consider a model of liquid crystals, based on a nonlinear hyperbolic system of differential equations, that represents an inviscid version of the model proposed by Qian and Sheng. A new concept of dissipative solution is proposed, for which a global-in-time existence theorem is shown.

The dissipative solutions enjoy the following properties:

(i) they exist globally in time for any finite energy initial data;

(ii) dissipative solutions enjoying certain smoothness are classical solutions;

(iii) a dissipative solution coincides with a strong solution originating from the same initial data as long as the latter exists.

László Székelyhidi (University of Leipzig)

Convex integration in fluid dynamics

In the talk we present the technique of convex integration for constructing weak solutions to various equations in fluid mechanics. We will focus on the recent resolution of Onsagers conjecture, but also discuss further directions and in particular the applicability to dissipative systems.

Károly J. Böröczky (CEU and Renyi Institute)

On the L_p dual Minkowski problem

Joint work with Ferenc Fodor (University of Szeged)

The q^{th} L_p dual Minkowski problem is a recent far reaching generalization of the classical Minkowski problem (the case $p = 1$ and $q = n$) solved by Minkowski, Alexandrov, Nirenberg and Caffarelli. The q^{th} dual curvature measure was defined in the 2016 Acta Mathematica paper by Huang, Lutwak, Yang, Zhang, and the L_p version is proposed in the 2018 Advances in Mathematics paper by Lutwak, Yang, Zhang. The case $p > 0$ and $q < 0$ is solved independently by Huang, Zhao and Gardner, Hug, Xing, Ye, Weil. This talk focuses on the case $p > 1$ and $q > 0$, including a discussion on the regularity of the solution.